

DYNAMIC RESPONSE ANALYSIS FOR DEVELOPMENT OF FLEXIBLE LIGHTWEIGHT VEHICLE CHASSIS USING CAE TOOLS

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ABSTRACT

This paper is elaborated the works done in studying dynamic response analysis of flexible lightweight vehicle chassis using CAE tools. The dynamic response analysis is implemented to analyze the lightweight material as pursue the automaker's goal in making of the lighter vehicle. Light weighting is the concept that being used by an automaker in order to overcome the fuel consumption and performance. This analysis is developed by comparing the conventional structure which is steel and aluminum as an alternative material which is lighter than steel, as one of the ways in reducing the weight of a vehicle. This simulation model is designed with the geometric and conditions similarity for both types of material to develop the accurate analysis of dynamic response analysis of lightweight vehicle chassis. The study on the chassis is developed on the go-kart model. The simulation model of go-kart is developed as a multi-body model which is consisted of four tires linked to the chassis. The tires receive the disturbance force from the road and transmit to the chassis. Chassis is the important structure that carried the entire load attached to the vehicle such as, engine system, braking systems and steering systems. Chassis structure also carried the driver who is affected by vibration. As the go-kart model is designed without suspension systems, thus, the chassis structure can produce high level of vibration from the little source of disturbance. This vibration phenomena would effect on human health if exceeds the limit of endurance. The effect of the vibration of the structure is measured in acceleration, velocity and displacement. To validate the simulation model, the result of the physical model is used. Through this dynamic response analysis, indicating the steel chassis produces high levels of vibration rather than aluminum chassis. This simulation analysis method has described the ability of CAE in order analyzed the anticipated disturbance during the design stage of lightweights vehicle.

KEYWORDS: CAE, Simulation, Lightweight Vehicle Chassis, Vibration Analysis

Article History

Received: 21 Feb 2018 | Revised: 13 Mar 2018 | Accepted: 19 Mar 2018

INTRODUCTION

In this modern age, transportation or vehicles become a necessity in daily life. Various vehicle designs and function is manufactured by the manufacturer. The automotive industry is growing rapidly and manufacturers are competing for each other to produce good quality of vehicles. There are satisfaction criteria of a vehicle, such as the economic fuel, high performance and comfort. In order to develop economic fuel and high -performance vehicles, lightweghing becomes the idea for an automaker to keep making the better car. The vehicle weight is reduced by using the alternative material such as aluminum. For the lightweight material, vibration is one of the issues to satisfy the comfort of the vehicle consumers [1].

Go-kart vehicle model is developed to study the effect of vibration on the vehicle, due to weight reduction. This go-kart vehicle is a simple constructed system, and concentrated in the design of the chassis structure due to weight limitation. The design of the chassis needs to be strong enough to support all the weight attached to the chassis. Therefore the chassis structure is the important component that will sustain the vibration transmission from the disturbance force such as road surface, and others flexible part. While the elastic behaviors of the structure are required due to the absent of suspension systems. Undamped structures can produce high levels of vibration from low level disturbance sources when frequency disturbance is close to one of the natural frequencies of the systems. This means that the component with good design and low vibration can produce other problems when assembled the entire component. In order to avoid these problems at the design stage, it is necessary to model the system accurately and analyze its response to anticipated disturbances.

The dynamic behavior of the go-kart chassis is influenced by stiffness and geometrical design of the chassis and the tires properties [2]. Besides optimizing the tires designs, efficient design of the chassis structure also is the way to dissipate the vibration [3]. This can be practiced with an application of CAE software which is noticeable as a modern technique that saves time and money in developing a virtual model of the analysis. The application of CAE software becomes a really helpful tool when applied with correct skill and expertise to model the simulation with the software virtue. The designers are able to design and simulate the model virtually and minimize prototyping construction in the production process. There have been many analysis performed in CAE to study the performance of the vehicle chassis such as a multi-body model designed in ADAM software by G.Mirone for studying dynamic performance of vehicle at different speed and steering wheel angle [4] and the applied finite element method in ANSYS software by Harshal Patil [5] to study analysis design of go-kart through finite element method. Biancolini [2] in his research also developed a simulation model to study dynamic behaviors of chassis go-kart. Accordingly, this paper emphasizes an analysis of dynamic loading on vehicle chassis performed by designing the vibration phenomena on the chassis go-kart when hit the road bump surface. A virtual multi-body dynamic model of the go-kart is constructed through the abilities of CAE ABAQUS software and analysis is designed base on the real physical model [6][7].

ASSEMBLY MODELS

The condition of a lightweight vehicle rolling on the road bump is modeled virtually through CAD and CAE software. The go-kart vehicle is built in a few body that is linked together to develop a multi-body dynamic model. The multi body model in Figure 1 is including 2 front tires and 2 rear tires that are linked to a chassis body. This model is developed by This model is developed by referred to the real physical model in Figure 2.

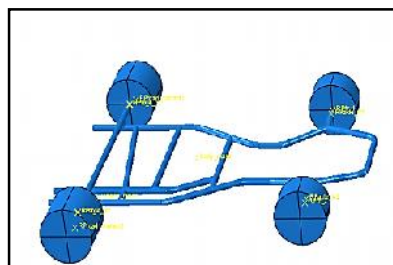


Figure 1: Assembly Models of G-Kart Vehicles



Figure 2: Real Models of G-Kart Vehicles

Chassis structure is developed as wireframe features and defines the profile shape as tubular in the CAE ABAQUS [9]. The profile shape for the chassis is described by thickness and radius of the tube structure. Meanwhile, tires are designed as a solid body without a others component such as thread band, wheel ring, and pressure air. The tires are designed base on mass distribution study in previous work [10].

There are some simplification and assumption made in the model built to focus and pursue the target of the study. In this study, the go-kart vehicle is model to moving straight and hit the road bump for front and rear tires. In this situation, the cornering phenomenon has not occurred, and the torsional of the structure, steering and braking systems is not included. The engine systems and the driver are presented through the point mass linked to the chassis. The points mass are representing the circumstance of load distribution through chassis structure and tires [10]. The process to develop the model is using integration of CAD and CAE method [9]. Each part of multi-body is designed in CATIA software then transferred to the CAE ABAQUS software for simulation. The designed parts are assembled and linked together in CAE ABAQUS. Connectors are used to join between tires and chassis that defined the degree of freedom for each part. The connectors which are assigned are allowed the required motion for moving the go-kart model. The parts are designed as a flexible body to study the dynamic response of the chassis in this analysis simulation.

Meanwhile, weights and dynamic behavior of the designed parts are given by the material properties that are assigned to the bodies. Chassis model is using AISI steel property which is 7800kg/m^3 for density, 0.3 of Poison Ratio and 270GPa for elastic modulus. The tire models are using rubber properties which is 1200kg/m^3 [11]and elastic modulus are obtained from the experimental radial stiffness value in the others research [2].The Young modulus for tires is derived from the radial stiffness of tire through the equation 1. This equation is known as specific stiffness or stiffness-to-weight ratio which is stiffness driven. The radial stiffness of the tires are obtained from the previous work. Through to the radial stiffness, the elasticity of the material is defined to model the tires.

$$\text{Specific stiffness} = \frac{E}{\rho} \quad (1)$$

E = Young Modulus

= density

Other than the given density, the weight of the part in the virtual model are also depends on the size of the design bodies. All in all, the weight of the simulation model resulted from multi-body joined in table 1

Table 1: Mass of Assembly Model in the Simulation

Instance Body	Quantity of Instances Body	Mass properties (kg)
Chassis	1	11.38
Front tire	2	5.47 x2
Rear tire	2	8.21 x 2
Point mass of engine	1	30
Point mass of drivers	1	50
TOTAL	7	118.75

A few models from the certain study also designed with varied weight such as 97 kg for overall weight of go-kart model in the journal of Harshal Patil [5], 150 kg for the models weight including the driver for the model of C.Ponzo studied [10], 140kg is the weight of the model in G.Mirone study [4], and 180kg for the go-kart model presented in go-kart championship final report by Prashan Tiwari and teams [12]. Each of the models studied comes with different design of the chassis structured either differing in chassis design, material or size of the structure.

MULTI BODY DYNAMIC MODEL

The assembled model is designed the mechanism to build mobility of assembly. The mechanism is consisted links and joints to transfer the motion of the source to the target and cause the possible motion. The motion between the bodies is recognized as a kinematic study while the outcomes of force exerted on the bodies during motion are the kinetic study. Both kinematic and kinetic study forms the dynamic analysis. It is related to the kinematic when the initial velocity is prescribed on the chassis go-kart to produce required motion. The chassis response of the go-kart velocity is necessarily a kinetic study. This multi-body model of go-kart vehicle is using a lumped mass approach by shrink the complex parts into a single element such as the design for tire models. Other tires component such as air pressure, inner thread and rim are simplified into the single solid model. Tires and all the assembly part need to be defined the degree of freedom. Each part in the assembly model contains 6 degrees of freedom. Road profile is the source of disturbance force of the moving tires in this analysis and tires linked to the chassis through connectors. The relation of all parts is summarized in Table 2.

Table 2: Description of Degree of Freedom for the Assembly Model

Parts	Quantity	Initial d.o.f	Constraint	Final d.o.f
Road Profile	2	2(6) =12	x,y,z, Rx,Ry,Rz 2(6) = 12	12-12 = 0
Tires	4	4(6) =24	z, Rx, Ry, Rz 4(4) = 16	24-16 = 8
Joints	4	4(6) =24	x,y,z, Rx,Ry, Rz 4(6) = 24	24-24 = 0
Chassis	1	1(6) = 6	z, Rz 1(2) = 2	6-2 =4
Mass of Driver	1	1(6) = 6	z, Rz 1(2) =2	6-2= 4
Mass of Engine	1	1(6) = 6	z, Rz 1(2) = 2	6-2= 4
Total	13	78	58	20

ANALYSIS SIMULATION DESIGN

The designed bodies of the go-kart are assembled and position on the road surface in order to perform dynamic response analysis on the chassis for the multi-body dynamic model in Figure 3.

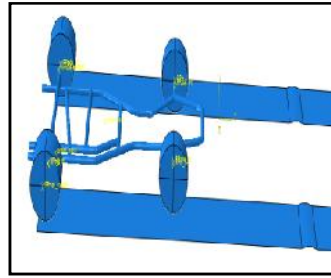


Figure 3: Simulation Model

Road profile is created and described the interaction between tires and road surface. For the initial condition of the model, a boundary condition is applied to all body and velocity is defined on the chassis to make the go-kart moving along the road surface. Besides, a constraint is used to build and precede the simulation with the correct degree of freedom for each part involved in the model.

The material properties are assigned to the deformable body which is , chassis frame and tires. The material used for chassis body is steel. While the properties of the tire is using radial stiffness in previous work[10] to obtain the elasticity of the tires. The go-kart model in the simulation is simulated at the speed of 20km/h while the road bump height is 10mm which is equal to experimental condition. Time of running the simulation is assigned until the front and rear tires of go-kart model is moving over the road bump model. The result is extracted from the chassis structure such as described in Figure 4.

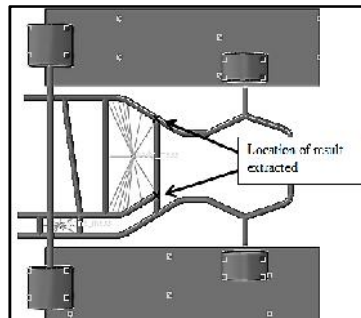


Figure 4: Position of Point for Extract the Result of History Output

NUMERICAL APPROACH OF THE SIMULATION MODEL

This simulation model is built up as a nonlinear dynamic model. There is non-linear material for stiffness and elasticity granted in the simulation model and motion assigned to the model. The non-linear system is leading to uncertainty and unpredictable output. The output condition is not expected to be directly proportional to the input. Basically, the equation 2,3,4 are equations of a degree of freedom for the go-kart moving for each plane [2] describe in Figure 5.

$$\sum M_y = I \ddot{\theta} \quad (2)$$

$$\sum F_x = m \ddot{x} \quad (3)$$

$$\sum F_z = m \ddot{z} \quad (4)$$

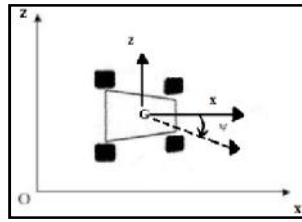


Figure 5: Degree of Freedom in 3 Plane

The simulation model is developed in 3D approach with 3 axes. According to the requirement in this analysis designed, a degree of freedom is reduced by the neglected degree of freedom in a yaw direction (ϵ). The model is designed with vertical reaction force is on the y-direction. The tractive force is transmitted to the vehicle chassis structure from the road surface through the tires. This go-kart model is designed with the absence of the suspension so the tire stiffness (k) are considered. The input to the tires is defined is from the road profile. The model is described in Figure 6 for whole model and Figure 7 for quarter vehicle model.

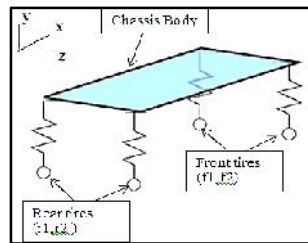


Figure 6: Load Transfer Model for Whole Vehicle

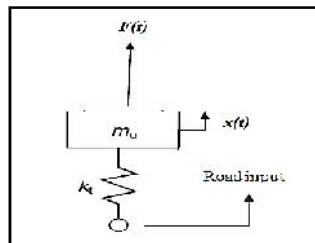


Figure 7: Quarter Vehicle Model

$$R_{f1} + R_{f2} + R_{r1} + R_{r2} - mg = 0 \tag{5}$$

The spring is represented both tires and chassis stiffness. The equation of equilibrium in equation 5 is for static a condition to portray the weight and load acting on the tires. The chassis is linked and responds to the tire stiffness. The simulation is modeled as a Go-kart model transverse on the road bump surface. The vertical load for this model is a symmetrical load when the two tires on an axle hit a symmetrical bump simultaneously. The structure is assumed as a symmetrical beam. The road bump causes the model move up and down perpendicular to the road surface and produce high vertical acceleration. The loading case during the vehicle hit the road bump is described in Figure 8.

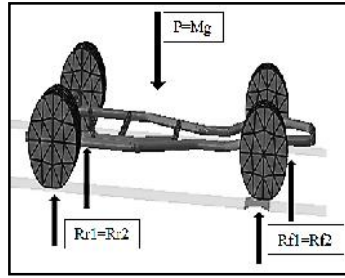


Figure8: Vertical Loading of the Model

The dynamic behavior of the chassis structure is modeled by simulating the go-kart vehicle passing over the road bump. The results of dynamic models are executed from the equilibrium of applied forces and the rate of change of momentum. This non-linear analysis is consigned the Second Law of Newton, which is forces are proportional to the acceleration multiple with the mass of the systems. The finite element system is applied the second order of differential equation 6 and the solver method selection available is an implicit and explicit method [8].

$$M a + C v + K x = F(t) \tag{6}$$

$$M \ddot{x} + C \dot{x} + K x = F \tag{7}$$

The equation (6) and (7) is described as **M**= Mass, **C**= Damping force, **K** = stiffness, and **F** = External force. While, \ddot{x} , \dot{x} , x is represent as acceleration, velocity and distance.

This non-linear equation is used to satisfy the random behavior and unpredictable of the dynamic response systems. Since the initial condition of the model is defined, the calculation of the analysis is involved with the configuration of the model moving forward relative to time through the integration the equation of motion. The finite difference equation is used, where are

$$v = \frac{du}{dt} = \frac{\Delta u}{\Delta t} = \frac{u_j - u_i}{t_j - t_i} \tag{8}$$

$$a = \frac{dv}{dt} = \frac{\Delta v}{\Delta t} = \frac{v_j - v_i}{t_j - t_i} \tag{9}$$

$$u = \frac{dl}{dt} = \frac{\Delta l}{\Delta t} = \frac{l_j - l_i}{t_j - t_i} \tag{10}$$

In equation 8, 9, 10, 'i' represents the initial condition of the model, while 't' in the initial state is equal to 0. When at the initial condition $v_{t=0}$ is known then u_j is able to be solved. Substitution in the equation of equilibrium can be used to solve for acceleration. The same method is to be used when t_j is known.

This equation is involved with discrete time increment (Δt) that specified the variation of output. To counter the complexity of the finite element model, dynamic explicit time integration method is used as the solver. This numerical integration scheme is used because it is an efficient computational method for large models with a relative dynamic response and contains general contact definition. Numerical explicit time integration is implemented through the central difference method

$$M^t \ddot{U} + C^t \dot{U} + K^t U = {}^t R \tag{11}$$

$$\dot{U} = \frac{1}{2\Delta t} ({}^{t+\Delta t} U - {}^{t-\Delta t} U) \tag{12}$$

$$\ddot{U} = \frac{1}{(\Delta t)^2} ({}^{t+\Delta t}U - 2 {}^tU + {}^{t-\Delta t}U) \tag{13}$$

The equation of 11, 12 and 13 are described \ddot{U}, \dot{U}, U is representing a vector of acceleration, velocity and distance respectively. The equilibrium equation 11 is used to solve an equation for 't+ t'. This finite element approach is used to extract the result at the selected location of the chassis structure.

EXPERIMENTAL RESULTS

The experiment is conducted by using a physical go-kart model that is used to build the simulation model. Vibrations are measured by using the accelerometer and interpret through data acquisition software. The method applied is a Data Acquisition (DAQ) process through Laboratory Virtual Instrument Engineering Workbench (Lab-View) from NI (National Instrument). The DAQ process flow is described in Figure 9.



Figure 9: DAQ Process Flow

Generally, data acquisition process is involving the process of acquiring a signal from the real-world mechanical phenomena, digitizing the signals, analyzing and visualizing the data. To develop a consistent result and acceptable result, the experiment is repeated for several times. Thus, in this research, 6 sample data is collected for the refining experimental result. In this experiment, the measured value is on the gravitational direction or vertical axis of the go-kart. The measured data from the accelerometer is listed in the following table 3.

Table 3: Result of Physical Model

No of Experiment	Result (m/s ²)
1	3.30
2	2.88
3	2.97
4	2.97
5	3.07
6	2.80
Average	3.00

Since the acceleration value for vibration is immeasurable, RMS value is figured out using LABVIEW application. Result of the RMS value. The result obtained in table 2 is fluctuated due to uncontrollable speed consistency. The go-kart is driven equal to the velocity in the simulation which is 20km/h. However, it is difficult to maintain the speed of the go-kart in the experiment without the speed control device and the capability of the driver to handle others measurement tools during driving. Due to that condition, the acceleration result has fluctuated in a certain range and the value is determined by average quantity. A result from physical testing is used to validate the simulation model constructed in the CAE ABAQUS software.

SIMULATION RESULTS

Result of simulation analysis in ABAQUS software is claimed in acceleration, and velocity graph. These outputs are extracted at the designated point on the chassis model equal to experimental condition, which is under the drivers' seat. The result is shown in figure 10 and 11.

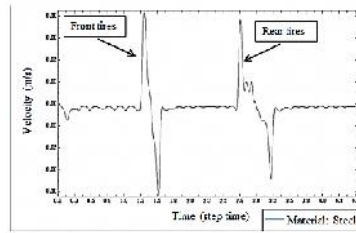


Figure 10: Velocity (m/s) vs Step Time

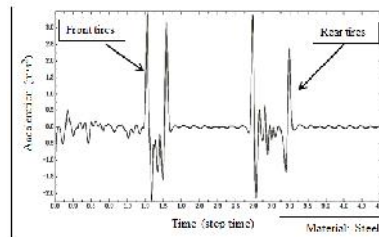


Figure 11: Acceleration (m/s²) vs Step Time

The result in a simulation is clearly shown the two peaks plotted in the graph during the front and rear tires is hit the road bump. The pattern of the plotted graph shows that the acceleration of go-kart model has a high value when the tires start hitting the road bump, decrease when on the tires is on top of the bump and increase again when the tire rolling down the bump. This describes the relation of the velocity and acceleration extracted from the chassis. The result for each acceleration and velocity output is summarized in table 4.

Table 4: Result of the Steel Structure in Simulation Model

	Front tire	Rear Tire
Acceleration (m/s ²)	3.46	3.37
Velocity (m/s)	0.10	0.09

VALIDATION RESULTS

This model is validated through the quantity agreement between the acceleration value in experimental testing and simulation analysis. In the simulation, the result can be obtained for front and rear tires, while the value of the experiment which is identified by using RMS value. By using the average value for both experiment and simulation, the result is compared in table 5.

Table 5: Comparison between Simulation and Experiment

Simulation Result (m/s ²)	Experimental Result (m/s ²)	Percentage Error (%)
3.41	3.00	13.5

This percentage error is to clarify the capability of the simulation model to perform an analysis problem for the aluminum chassis of go-kart vehicle.

ANALYSIS OF LIGHTWEIGHT MATERIAL

By using the same design of the steel chassis in the simulation analysis, the material of the chassis is changed to the aluminum by according to the properties in table 6.

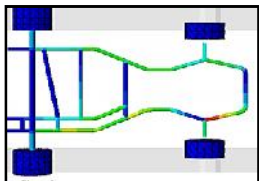
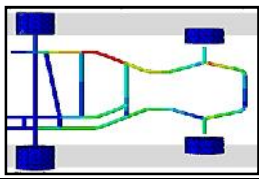
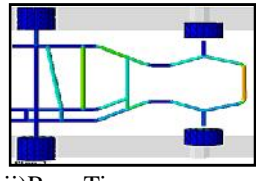
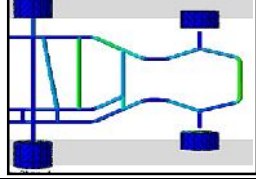
Table 6: Aluminum Properties

Aluminium Properties	Value
Density	2700kg/m ³
Yield Modulus	70GPa
Poison Ratio	0.33

The changes of the chassis material from steel to aluminum caused mass of the whole model is reduced. The initial velocity is defined for 54km/h, which is same for both models. Meanwhile, same load position and value are also attached equally for both chassis model. This simulation model is used to compare the performance of the steel and aluminum chassis.

The material properties of aluminum assigned to the simulation model caused the analysis time increased and larger space of the processor is consumed rather than using steel properties. On the other hand, the comparison of steel and aluminum structure is seriously discussed according to strength, weight, stiffness and elasticity. The result is presented in the table 7.

Table 7: Comparison Analysis Result between Steel and Aluminum Chassis

Steel Structure	Aluminium Structure
Von Misses Stress i) Front tire  ii) Rear Tire 	Von Misses Stress i) Front tire  ii) Rear Tire 
Accelereation(m/s²) i) Front tire= 11.99 ii) Rear Tire = 11.32	Accelereation (m/s²) i) Front tire= 4.44 ii) Rear Tire = 4.54
Velocity (m/s) i) Front tire=0.41 ii) Rear Tire = 0.37	Velocity (m/s) i) Front tire=0.14 ii) Rear Tire= 0.15

The differences result between steel chassis and aluminum is described in the graph plotted in Figure 12 and 13. The graph patterns obviously show the step time of the hitting road bump occurred. Both acceleration and velocity value is increased when the tires hit the road bump.

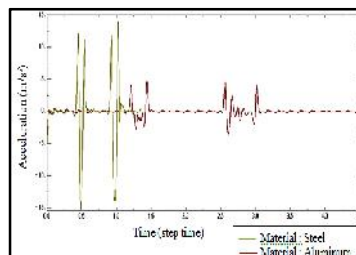


Figure 12: The Comparison Graph for Acceleration Result

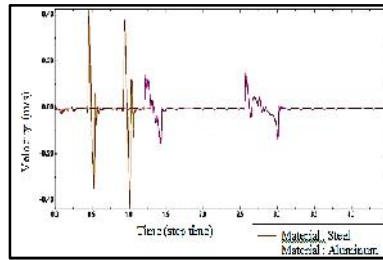


Figure 13: The Comparison Graph for Velocity Result

Meshing process for aluminum takes longer time than steel structure design. Consequently, time for aluminum is extended for 4.5 step time to perform the complete analysis compare to step time needed for steel which is 1.45 step time as shown in figure 12 and 13. Other than step time, both models are in the same condition, such as velocity, driver and engine mass, and the design of the chassis and tires properties.

As well-known, Von Mises Stress is used to predict the design safety due to loading condition. It is the value before the structure starts yielding at any point of failure when over the elastic limits. From the comparison table, the counter plotted of Von Mises Stress of steel got higher value rather than aluminum. This explained that aluminum structure got less yield strength and easily yielding than the steel structure. The quantity scale of Von Mises Stress is expressed in colors in the finite element model. By undergone the prescribed condition, both models showed the different result of stress distribution. In the comparison graph, the model of steel structure produced higher peak for acceleration and velocity compared to the aluminum structure. This is explained that steel structure is oscillating more than aluminum structure when the vibration is excited by the road bump.

These result, Von Mises stress, and acceleration graph describe the aluminum and steel application due to the dynamic performance of chassis structure. The steel structure is a high strength structure and very stiffness to compensate the vibration effect while aluminum structure has high elasticity and ability to reduce the vibration effect. Stress distribution result shows that steel structure is more safety to the driver but it is less appropriate for dynamic behavior. Although, the properties of the aluminum which is easily deformed is not safe to a driver through the analysis simulation the design of aluminum structure is can be optimized.

CONCLUSIONS

The simulation analysis has been studied and improved according to the experimental testing comparisons. The model of simulation is developed as a multi-body model which is each part is linked together and obtains the appropriate analysis result. The use of analysis software is the modern technique that offered an assistance to analyze the problem which has a few advantages such as reduces cost and time at the design stage of the product development. The simulation model with the sufficient accuracy will expand the ability of a designer to solve the complex problems. Through the research implementation, there are some knowledge and skills that are improved by applying the CAE ABAQUS. The influenced parameters in simulation model are recognized through result visualization. Analysis inputs are easily controlled such as the value of velocity, load, and material properties. Meanwhile, contact and joint definition is required more analytical skill to define. The result of this simulation analysis is affected when the parameters changed. Proper analysis result can be obtained when the correct inputs are applied.

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